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PROBLEM OF THE INTERACTION BETWEEN A SUPERSONIC FLOW AND A CLOUD OF PARTICLES

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The wave pattern which occurs when shock waves interact with clouds of particles is of considerable importance when investigating the ignition of reacting gas suspensions, the rising of dust behind the leading shockwave front, the damping of shockwaves by dust suspensions etc. These flows, depending on the initial parameters, can be reduced, in a rough approximation, to the following [1-3]:

– motion with a collective shockwave which stays (moves) in front of the cloud of particles or is attached to its leading edge (mode 1),

– the flow of a gas suspension, each particle of which has an individual shockwave (mode 2).

A simple quantitative estimate was derived in [4, 5] of the maximum Mach number M_2 of the continuous phase behind a shockwave front which as passed through a particle (its velocity at the initial instant is zero):

$$M_2 = 2(M_0^2 - 1)\{[M_0^2(\gamma - 1) + 2][2\gamma M_0^2 - (\gamma - 1)]\}^{-1/2}.$$

The results of calculations using this formula, presented in the table, show that when $M_0 > M_0(\gamma)$ the absolute velocity behind the shockwave front will be supersonic and an individual shockwave is formed in the region of each particle (M_0 is the Mach number of the oncoming shockwave and γ is the adiabatic exponent of the gas).

According to the representations described briefly in [1] a common or collective shockwave can be formed in front of a certain pair of individually chosen particles when the transonic shock layers overlap. The criterion for this to form can be formulated in terms of λ, λ_* ($\lambda = l/d$, where l is the mean distance between the bodies, d is the diameter of a particle and the asterisk denotes parameters of the transonic zone) in the form

$$l/d < l^*/d. \tag{1}$$

As the particle accelerates the relative Mach number falls, and when $M_2 \leq 1$ the individual shockwave in the region of each particle disappears. An analysis of the wave pattern of supersonic flow around a cloud of particles was carried out in [1] using the results of calculations on the supersonic flow around ellipsoids [6], and it was concluded that one should not expect the occurrence of a collective shockwave under the conditions described in [2].

Results of experiments on the flow around a collection of particles of bronze (d = 80 μm), and magnesium (d = 300 μm) were given in [7], and showed that the transition from mode 2 to mode 1 occurs with an increase in the volume concentration of the particles. To interpret the experimental results the criterion (1) was then used, reformulated in terms of the volume concentration of the particle m_2, and it was concluded that there is a certain limiting value of $m_2 = m_*$ which, if exceeded, leads to the occurrence of a collective shockwave.

A detailed investigation of the aerodynamic interference of the particles of a cloud when acted upon by a supersonic flow which occurs behind a passing shockwave is therefore of interest.

Formulation of the Problem. Suppose we have a cloud of particles situated in space and which is finite along the direction of one of the axes. We will investigate the problem of the incidence of a shockwave (SW) on the front of the cloud, which will be modeled by a set of particles (small plates), a body with a sharp leading edge, elongated in one direction (Fig. 1).

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TABLE 1

γ	M_0			
	2,0	2,5	3,5	4,5
	M			
1,35	1,01	1,26	1,57	1,73
1,4	0,96	1,20	1,47	1,61
1.45	0.92	1.14	1.39	1.51

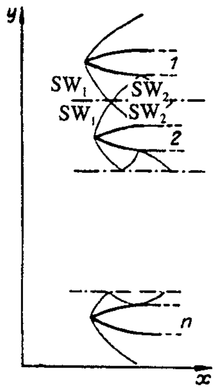

Fig. 1

We will analyze the flow pattern which occurs when flow occurs around the small plates 1 and 2 situated at the upper edge of the cloud.

In the case of supersonic flow an additional jump occurs behind the incident shockwave ($M_2 > 1$) at the particles with a sharpened edge. On the whole a complex interference (interaction) pattern occurs between the shockwave itself and the surface of the body.

The resulting aerodynamic force acting on a particle is given by the expression

$$\mathbf{F} = \int C_p(x, y, z) d\mathbf{S}.$$

Here the integration is carried out over the surface of the body. In a system of coordinates connected with the body, $\mathbf{F} = Xi + Yj + Zk$, where X is the component of the drag, ignoring viscosity, Y is the component of the lifting force, Z is the component of the side force, dS is the vector of the element of area of the body surface, $C_p = \Delta p/q$ is the pressure coefficient, and $\Delta p = p - p_0$, p, p_0 are the instantaneous pressure and the pressure of the incoming flow, and q is the velocity head. The Y axis is directed into the inner side of a particle (the inner side is the surface of the particle facing its neighbors).

In the central part of the cloud, i.e. for particles which have neighbors on all sides (particle 2), the flow pattern and the interaction with the shockwave are symmetrical about the axis of particle 2. In this case the distribution of the pressure coefficient over the surface of the body is a function of x, and in each transverse cross section of the body $C_p = C_p(x) =$ const, so that the components of the forces $Y = \int C_p dS_y = 0$, $Z = \int C_p dS_z = 0$ and the resulting force is directed along the x axis.

On the side edge of the cloud, when there are no neighboring particles on the external side of a periphery particle, the flow pattern and the interaction with the shockwave ceases to be symmetrical. In this case the system of incident and reflected waves produces zones of increased pressure only on the inner surface of the particle. A flow pattern similar to that presented in [8, 9] is obtained.

It is obvious that the repulsive force Y depends both on the number of zones of increased pressure and on the intensity of the pressure in these zones.

For the case when the distance between the particles is comparable with their diameter, the number of zones of increased pressure is determined by the integer part of the quantity

$$K = L\mathrm{tg}(\beta)/(l - d). \qquad (2)$$

Here L is the length of particle and β is the angle of inclination of the shockwave.

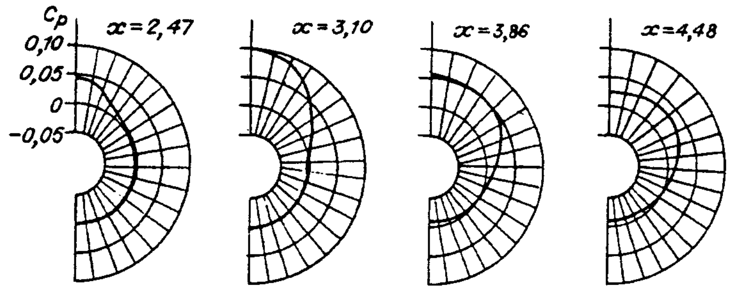

Fig. 2

For K < 1, the head shockwave intersects the axis of neighboring particles behind the stern part. There is no supersonic interference. Flow occurs around a particle as an isolated body. When K > 1 supersonic interference between the particle occurs.

It should be noted that the criterion K holds when there is regular interaction both between the head jumps and the internal jumps for regular reflection from the surface of the body. However, depending on θ_k (the angle of inclination of the leading edge of a small plate to its axis) and M_0 for a fixed distance between the bodies, irregular reflection and interaction with the shockwave is possible. This may lead to the formation of a Mach configuration and the occurrence of a collective shockwave. This indicates that for a fixed particle concentration a change in their local geometrical parameters may also lead to collective shockwave.

We will dwell in more detail on the mechanism by which dispersion of the leading edge of abounded cloud of particle occurs. For our purposes we are interested in the pressure distribution pattern on the surface of the first periphery of the small plate and different sections downstream as a function the azimuthal angle φ ($\varphi = 0$ is the inner side and $\varphi = 180°$ is the outer side of a particle with respect to the plane of interaction lying between the bodies; in a cylindrical system of coordinates the X axis is directed along the axis of the body). The pressure distribution pattern over the body depends on the number of interacting shockwaves and rarefaction waves with its surface and is determined by (2). The degree of compression of the flow depends on the intensity of the incident shockwaves and the local angle of incidence to the surface of the body.

In Fig. 2 we show the distribution $C_p = (p - p_0)/q_0$ for different sections x for $M_0 = 4.017$ and $\varphi \in [0 - 180°]$, obtained by numerical solution of the problem of the spatial supersonic interaction between the bodies [9].

In a system of coordinates connected with the particles, the Y axis is directed towards the external side of the interacting particles. The quantity ΔY represents the forces acting on a defined element of area (particle) with planes x = const, x = const + Δx, and is determined by the derivative (gradient) of the pressure coefficient C_p along the φ direction.

In particular, when $\partial C_p{}^p/\partial\varphi < 0$ $\Delta Y > 0$ repulsive forces act on the particle, and when $\partial C_p/\partial\varphi > 0$ $\Delta Y < 0$ attractive forces act on the particle. It turned out that under the flow conditions investigated in [9] repulsive forces were obtained.

Experiments were described in [10] to determine the forces of interference between two solids of rotation with ogival and conical head sections. They also showed that there were repulsive forces present over a side range of distances between the bodies l and the extended cylindrical part λ_c. It should be noted that forces of attraction occur in these experiments [10] over a very narrow range $(R, \lambda_c) \in (1.1; 1.3) \times (3.0; 5.0)$. This enables us, the first stage of the investigation, to neglect the effect of the repulsive region.

When repulsion exists between the first and second particles, particles 1 and 2 will separate due to the action of the shockwave SW_1 (Fig. 1). This may serve as one of the possible mechanism by which the cloud disperses due to the action of the shockwave.

We will use the above results to analyze experiments on the interaction between a shockwave and a cloud of particles.

Problem 1. We have a compact cloud of particles of plexiglass on which the shockwave is incident ($M_0 = 4.7$, $\Delta t = 40$ μsec, Fig. 4.4 from [11]). In the first photograph one can see the shockwave incident on the cloud. The next ones demonstrate how the leading edge of the cloud becomes gradually sharpened. there are obviously two processes occurring here. The first consists of the fact that, due to the aerodynamic interaction that occurs, the peripheral particles become detached from the nucleus and are then causes an additional lifting force due to flow around the particles at the angle of attack.

Problem 2. Suppose the shockwave glide along the walls of the channel so that a rise of dust occurs behind its front.

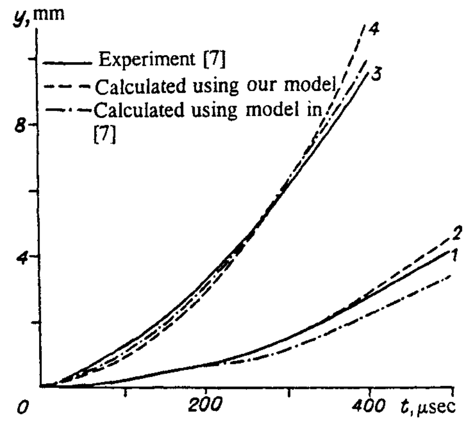

Fig. 3

This problem has attracted the attention of numerous researchers who use different hypotheses regarding the forces acting on a particle to explain the mechanism of the rising of dust.

A brief review of these publications can be found in [11]. The investigation has largely been carried out using particles of the same type, when the field of flow of the gas is formed due to interaction between the shockwave and a boundary layer. An important feature of the mathematical model employed is the need to specify the nonzero vertical velocity v_y of the individual particles. As a rule, no analysis is given of the reasons for the recoil of particles when $t = 0$ and these mathematical models only predict the evolution of particles having a specified v_y^0 at $t = 0$.

The use of the Magnus force, which acts on a particle rotating in a flow of gas, was suggested in [10] to describe the rising of particles. It is assumed that the particle is in translational motion with velocity $v_s(t)$, which is communicated to it by the flow of gas moving with velocity v_g along the x axis. It is assumed that $C_d = 0.7$ and that ρ_g and v_g are constant quantities. This enables one, with certain constraints, to write the integrals of motion of a particle for both components of its velocity vector. An important constant of this mathematical model, moreover, is v_0 − the translation velocity of a particle up to the instant of collision with another, for which a certain quantitative estimate is given.

We will propose a single quantitative model of the rising of a particle, which is free from the need to specify the parameter v_0, which is difficult to monitor, keeping the above considerations in mind.

To represent the motion we will assume that a pressure $p_1 < p_2$ acts on the external side of the boundary particle (p_2 is the pressure acting on the inner side of the particle). The equation of motion of a particle then has the form [12]

$$m\frac{dv}{dt} = \Delta pS,\ \Delta p = p_2 - p_0 = 0.5\gamma p_0 M_0^2 C_p, \tag{3}$$

where $m = 4\pi r^3\rho_{22}/3$; r is the effective radius of a particle, ρ_{22} is the true density of the material, p_0 is the pressure in front of shockwave and $S = 4\pi r^2$.

The rising of a particle can be represented as the following conclusive process. Initially, a boundary particle is acted upon by an interference force which arises from the interaction with he shockwave. As a result of this force it is shifted to a position when $C_p(t_b) = 0$ (t_b is the time it reaches the state in which the repulsive force is zero). The second particle then plays the role of the first (the boundary particle) and the process is repeated once again. We will replace this kind of discrete process by a continuous one with a certain effective value of the coefficient C_p, obtained by averaging over the interval $t \in [0, t_b]$.

Note than, for a lighter particle, lifting to a height where there is no strong interaction occurs in a shorter time ($t_0 < t_s$), and hence he would expect that the mean value of C_p of light particles may be different from that of heavy particles.

Equation (3) should satisfy the Cauchy data

$$v(0) = 0. \tag{4}$$

The solution of problem (3), (4) can be written in the explicit form

$$y = at^2/2,\ a = 1.5\gamma p_0 M_0^2 C_p/(r\rho_{22}).$$

In Fig. 3 we show curves of the height to which particles of bronze are lifted (curves 1 and 2) and similar curves for plexiglass (curves 3 and 4) in a flow behind a gliding shockwave. As can be seen, when $C_{p_b} = 0.02$ there is satisfactory agreement with experiment. The choice of $C_{p_0} = 0.0108$ (plexiglass) also leads to reasonable agreement between the trajectories. This enables us to assert that the mechanism of the rising of dust, based on the aerodynamic interference of particles, describes important features of the phenomenon.

Hence, in this paper we have qualitatively analyzed the pattern of aerodynamic interaction of particles in a cloud. We have indicated the possibility that repulsive forces exist between the particles, and we have given a criterion for its existence. On the basis of these representations we have proposed a simple quantitative model of the rising of dust behind a shock-wave front, which satisfactorily describes experiments.

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